Coxeter polytope:
A polytope P in $X = S^d$, $\mathbb{E}^d$ or $\mathbb{H}^d$ is called a *Coxeter polytope* if all dihedral angles of P are integer submultiples of π.

- The group G_P generated by reflections with respect to the facets of P acts discretely on X, P is the fundamental domain of G_P.
- Moreover, a fundamental domain of any discrete reflection group in X is a Coxeter polytope.

The highest dimension faces of polytopes are called *facets*.

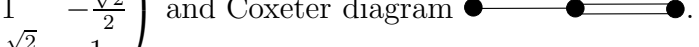
Gram matrix:
The *Gram matrix* of a Coxeter polytope P with n facets $f_1, \dots, f_n$ is a symmetric $n \times n$ matrix $G = \{g_{ij}\}$ such that

$$g_{ii} = 1, \qquad g_{ij} = \begin{cases} -\cos(\frac{\pi}{m_{ij}}), & \text{if } \angle f_i f_j = \frac{\pi}{m_{ij}}; \\ -1, & \text{if } f_i \text{ is parallel to } f_j; \\ -\cosh\ \rho_{ij}, & \text{if } f_i \text{ and } f_j \text{ diverge and lie at distance } \rho_{ij}. \end{cases}$$

Coxeter diagram:
It is convenient to represent Coxeter polytopes with *Coxeter diagrams*:

- **vertices** v_i correspond to facets f_i of P;
- **edges:**
  - v_i is joined to v_j by an edge labelled k if $\angle f_i f_j = \frac{\pi}{k}$;
  - if $k = 2, 3, 4, 5, 6$ one uses the following notation:
    - v_i is **not** joined to v_j if $\angle f_i f_j = \frac{\pi}{2}$;
    - v_i is joined to v_j by $(k-2)$-fold edge if $\angle f_i f_j = \frac{\pi}{k}$;
  - v_i is joined to v_j by a **bold** edge if f_i is parallel to f_j;
  - v_i is joined to v_j by a **dotted** edge if f_i and f_j diverge;
    - a dotted edge $v_i v_j$ is labelled by $\cosh\ \rho$, where ρ is the hyperbolic distance from f_i to f_j.

Examples:

- A spherical triangle with angles $(\frac{\pi}{2}, \frac{\pi}{3}, \frac{\pi}{4})$ is a 2-dimensional Coxeter polytope with Gram matrix $\begin{pmatrix} 1 & -\frac{1}{2} & 0 \\ -\frac{1}{2} & 1 & -\frac{\sqrt{2}}{2} \\ 0 & -\frac{\sqrt{2}}{2} & 1 \end{pmatrix}$ and Coxeter diagram  .
- A hyperbolic quadrilateral with angles $(0, \frac{\pi}{2}, \frac{\pi}{6}, \frac{\pi}{7})$ is a 2-dim Coxeter polytope with Gram matrix $\begin{pmatrix} 1 & -1 & -\cosh\ d_1 & -\cos\frac{\pi}{7} \\ -1 & 1 & 0 & -\cosh\ d_2 \\ -\cosh\ d_1 & 0 & 1 & -\frac{\sqrt{3}}{2} \\ -\cos\frac{\pi}{7} & -\cosh\ d_2 & -\frac{\sqrt{3}}{2} & 1 \end{pmatrix}$
and Coxeter diagram